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Kaluza, Klein, Confinement, and Nuclear Forces

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The nonsymmetric Kaluza-Klein and Jordan-Thiry theories are reviewed as interesting propositions of physics in higher dimensions. It is shown how a dielectric model of confinement can be derived from "interference effects" in these theories. It is postulated that the old puzzle of nuclear physics, σ-particles, can be connected to the skewon field $g_{[\mu\nu]}$ and the scalar field Ψ in the nonsymmetric Jordan-Thiry theory. Similarities are pointed out between the nonsymmetric Jordan-Thiry Lagrangian in the flat space limit and the soliton bag model Lagrangian. Finally the nonsymmetric Jordan-Thiry Lagrangian is proposed as the bosonic part of the strong interaction Lagrangian.

1. INTRODUCTION

There has been a recent revival of interest in the ideas of T. Kaluza and O. Klein on the geometrical unification of gravity (described by General Relativity) and other fundamental interactions using many-dimensional manifolds (five-dimensional in the original work by Kaluza) (Kaluza, 1921; Klein, 1926, 1939). Such an approach seeks the unification of two major principles in physics, local gauge invariance and local coordinate invariance, reducing them to the second in a 5-dimensional world. The additional dimensions cannot be directly observed. In the present approach, I propose a development of these ideas using the non-Riemannian geometry of Einstein's unified field theory (the so-called Einstein-Kaufman theory) (Einstein, 1945, 1951; Einstein and Kaufman, 1954; Einstein and Strauss, 1946; Kaufman, 1955, 1956).

In the Kaluza-Klein approach there were no "interference effects" between gravity and electromagnetism. This theory reproduces the Einstein

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and Maxwell equations in an already known form. In the non-Abelian Kaluza-Klein theory (which unifies the principles of local non-Abelian gauge invariance and local coordinate invariance) one faces a fundamental problem with the value of the cosmological constant. The cosmological constant predicted by the non-Abelian Kaluza-Klein theory is 10^{127} times greater than the upper limit from observational data (Kerner, 1968; Cho, 1975; Kopczyński, 1980). This forces us to abandon Riemannian geometry (the Levi-Civita connection) and to use non-Riemannian geometries defined on a multidimensional bundle manifold (a gauge manifold). Such geometries have been constructed (Kopczyński, 1980; Kalinowski, 1983a) and the cosmological constant disappears. Unfortunately, all of these approaches fail, in that they do not provide any "interference effects" between gravity (General Relativity) and Yang-Mills theory. They reproduce the Einstein and Yang-Mills equations in an already known form.

They could provide some "interference effects" between gravity and a gauge field (electromagnetic field) if the geometry is coupled in a covariant ("many-dimensional") way to the fermion field (described by many-dimensional spinors). In this way we can get some "interference effects" between gravity and a gauge field via interactions with fermions. Such approaches have been considered and one gets "gravitational-electromagnetic effects" (Thirring, 1972; Kalinowski, 1981a, 1984a), and in general "gravitational-gauge field effects." These effects are extremely small and cannot be observed using present experimental techniques. They provide a dipole electric moment for fermions, which results in P and PC breaking. Some problems connected to the minimal mass of a fermion in such a theory can be avoided by introducing a new kind of gauge derivative (Kalinowski, 1981b, 1982a, 1983b, 1984a).

However, none of these approaches can be considered as a true unification of gravity and other interactions. It seems that we have to deal with a change of notation. In some sense the five-dimensional Kaluza-Klein theory and its non-Abelian descendants are just "unified (many-dimensional) notations" for the Einstein and Maxwell (Yang-Mills) theory.

This does not mean that the problem of a notation is not important. Recall that the notation problem played a fundamental role in the construction of general relativity. One could not even imagine the invention of General Relativity without Minkowski space. Moreover, the Minkowski space notation of special relativity and the pre-Minkowski approach are equivalent [see, for example, Einstein's (1905) original paper]. This notation/language is a geometry. Probably the same holds in the construction of "a true unification" of gravity and other interactions in a Kaluza-Klein manner. We have an appropriate notation (a geometrical language—multidimensional language) and we should look for an appropriate geometry.

Recall that Minkowski space is a flat Riemannian manifold and the appropriate geometry of General Relativity is a curved Riemannian geometry.

Thus the present approach consists in finding such a geometry (such a Kaluza-Klein theory), and finding "interference effects" between gravity and gauge fields and their physical consequences.

The most interesting problem occurs when this new Kaluza-Klein theory is considered as a realistic model of strong interactions. Thus I consider this theory as the source of the classical dielectric model of confinement, supposing that the structural group $G = SU(3)_c$, and then adding spinor sources (quark fields). In this way the idea of confinement emerges from the physics in higher dimensions with a geometrical interpretation. The Lagrangian of the nonsymmetric Jordan-Thiry theory, in the flat space limit, resembles the soliton bag model Lagrangian.

In the next section I consider the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory as a proposal for the theory of strong interactions.

2. THE NONSYMMETRIC KALUZA-KLEIN (JORDAN-THIRY) THEORY

In the last few years a nonsymmetric Kaluza-Klein theory (Kalinowski, 1983c,d) has been constructed together with its extension to the nonsymmetric Jordan-Thiry theory (an additional scalar field connected to the effective "gravitational constant") (Jordan, 1955; Thiry, 1951a; Lichnerowicz, 1955). Non-Abelian extensions of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories have been found (Kalinowski, 1983e, 1984b).

It was possible to extend the nonsymmetric Kaluza-Klein theory to the case with material sources (including spin sources) and to include such phenomena as spontaneous symmetry breaking and the Higgs mechanism (with two critical points for a Higgs potential) (Kalinowski, 1982b, 1983f, 1984c). Thus it is possible to consider the "interference effects" between electroweak interactions (described by the geometrical version of the Weinberg-Salam-Glashow model) and gravity. Simultaneously, this allows us to build a more realistic model of Grand Unification, including the gravitational field.

The linear version of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories has been found (Kalinowski and Mann, 1983, 1984).

The first exact solutions in the five-dimensional (electromagnetic) case have been obtained (Kalinowski and Kunstatter, 1984; Mann, 1984). It was possible to find an extension of some earlier work to the case of the nonsymmetric Kaluza-Klein theory (Kalinowski, 1982c, d), i.e., an introduction of fermion sources leading to the small "interference effects" mentioned in Section 1 (a dipole electric moment for fermions and *PC* breaking).

This will be very helpful in finding a model of strong interactions, i.e., an extended QCD with "interference effects" between gravity and strong interactions.

The nonsymmetric Kaluza-Klein and Jordan-Thiry theories have a well-defined geometry on a multidimensional manifold [five-dimensional in the electromagnetic case and $(n+4)$-dimensional in the non-Abelian case, $n=\dim G$, where G is a gauge symmetry group]. The geometry in this theory is a geometry from Einstein's Unified Field Theory (Einstein, 1945, 1951; Einstein and Strauss, 1946) in the Kaufman version (Kaufman, 1955, 1956; Einstein and Kaufman, 1954). This version is known as the Einstein-Kaufman theory. In some sense this geometry is a multidimensional extension of the Einstein-Kaufman geometry. This geometry is defined on the gauge manifold (manifold of a principal fibre bundle) and is called the Einstein geometry. The nonsymmetric Kaluza-Klein (or Jordan-Thiry) theory is a generalization of the Kaluza-Klein (or Jordan-Thiry) theory and Einstein's Unified Field Theory.

These theories realize a true unification of gravitational and gauge fields in the following sense: they not only unify a local gauge invariance principle and a local coordinate invariance principle, but they provide "interference effects" between gravitational and gauge fields (electromagnetic field in the five-dimensional case) as well. One has the following "interference effects":

1. An additional term in the Lagrangian for the electromagnetic field equal to $2(g^{[\mu\nu]}F_{\mu\nu})^2$ [for a gauge field it is equal to $2l_{ab}(g^{[\mu\nu]}H^a_{\mu\nu})\times(g^{[\alpha\beta]}H^b_{\alpha\beta})$], where $F_{\mu\nu}$ is the strength of the electromagnetic field and $H^a_{\mu\nu}$ is the strength of the Yang-Mills field.
2. A new energy-momentum tensor for an electromagnetic field (gauge field).
3. Two field strength tensors for the electromagnetic (gauge) field, i.e., $F_{\mu\nu}$ and $H_{\mu\nu}$ ($=H^a_{\mu\nu}$ and $L^a_{\mu\nu}$).
4. The source in the second pair of Maxwell's (Yang-Mills') equations, i.e., a current $j_\mu(j^a_\mu)$.
5. The polarization of vaccum $M_{\mu\nu}=-(1/4\pi)(H_{\mu\nu}-F_{\mu\nu})$ $[M^a_{\mu\nu}=-(1/4\pi)(L^a_{\mu\nu}-H^a_{\mu\nu})]$ with an interpretation as the torsion in the fifth dimension (in higher dimensions in the Yang-Mills case).
6. An additional term in the equation of motion for a test particle (additional term for a Lorentz-force term in the electromagnetic case), as appears in the modified Kerner-Wong equation.

7. A dependence of the cosmological constant on a dimensionless constant μ with an asymptotic behavior

$$\rho(\mu) \sim \frac{\text{const}}{\mu^2} \qquad \text{for large } \mu$$

This constant in general is a rational function of μ, i.e.,

$$\rho(\mu) = \frac{P_m(\mu)}{Q_{m+2}(\mu)}$$

Thus it is possible to avoid some problems with the enormous cosmological constant that appears in the classical approach, when μ is chosen as a root of the polynomial P_m or becomes sufficiently large (Kalinowski 1983e, 1984b). The constant μ is simultaneously a coupling constant between a skewon field $g_{[\mu\nu]}$ and a Yang-Mills field in the linear approximation (Kalinowski and Mann, 1984).

In the case of the nonsymmetric Jordan-Thiry theory one gets some additional effects:

1. A Lagrangian for a scalar field Ψ.
2. An energy-momentum tensor for the scalar field Ψ.
3. Additional scalar forces in the equation of motion for a test particle (generalized Kerner-Wong equation).

The scalar field Ψ is connected to the effective "gravitational constant" by

$$G_{\text{eff}} = G_{\text{N}} \exp[-(n+2)\Psi]$$

where G_{N} is the Newton constant. This field seems to be massive, with short-range behavior (Yukawa-like behavior) (Kalinowski, 1982b, 1983d, 1984b; Kalinowski and Mann, 1983). In this way, there are no problems with the weak equivalence principle.

Some details of the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory follow.

Let P be the principal fibre bundle with structural group G, over space-time E with a projection π, and let us define on this bundle a connection w. Let us suppose that G is semisimple and that its Lie algebra g has a representation such that $\text{Tr}[(X_a)^2]$ is real and nonzero for every a. Here Tr is understood in the sense of the representation of the Lie algebra g (it is better to say in the sense of the representation of its enveloping algebra). On the space-time E we define a nonsymmetric, real tensor such that

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]} \tag{1}$$

$$g_{\alpha\beta} g^{\gamma\beta} = g_{\beta\alpha} g^{\beta\gamma} = \delta^{\gamma}_{\alpha} \tag{2}$$

where the order of indices is important. We also define X two connections on E:

$$\begin{aligned} \bar{\omega}^{\alpha}_{\beta} &= \bar{\Gamma}^{\alpha}_{\beta\gamma}\bar{\theta}^{\gamma} \\ \bar{W}^{\alpha}_{\beta} &= \bar{\omega}^{\alpha}_{\beta} - \tfrac{2}{3}\delta^{\alpha}_{\beta}\bar{W} \end{aligned} \tag{3}$$

where

$$\bar{W} = \bar{W}_{\gamma}\bar{\theta}^{\gamma} = \tfrac{1}{2}(\bar{W}^{\sigma}_{\gamma\sigma} - \bar{W}^{\sigma}_{\sigma\gamma})\bar{\theta}^{\gamma} \tag{4}$$

and $\bar{\theta}^{\alpha}$ is a frame on E.

For the connection $\bar{\omega}^{\alpha}_{\beta}$ we suppose the following conditions:

$$\begin{aligned} \bar{D}g_{\alpha+\beta-} = \bar{D}g_{\alpha\beta} - g_{\alpha\delta}\bar{Q}^{\delta}_{\beta\gamma}(\bar{\Gamma})\bar{\theta}^{\gamma} &= 0 \\ \bar{Q}^{\alpha}_{\beta\alpha}(\bar{\Gamma}) &= 0 \end{aligned} \tag{5}$$

where $\bar{D}$ is the exterior covariant derivative with respect to $\bar{\omega}^{\alpha}_{\beta}$ and $\bar{Q}^{\alpha}_{\beta\gamma}(\bar{\Gamma})$ is the torsion of $\bar{\omega}^{\alpha}_{\beta}$.

Let us introduce a natural frame on $\mathbf{P}$ (a horizontal lift base)

$$\theta^{A} = (\pi^{*}(\bar{\theta}^{\alpha}), \theta^{a} = \lambda w^{a}), \qquad 0 < \lambda = \text{const} \tag{6}$$

where $w = w^{a}X_{a}$ is a connection on $\mathbf{P}$. It is convenient to introduce the following notations. Capital Latin indices A, B, C run over $1, 2, 3, \ldots, n+4$, $n = \dim G$. Lower case Greek indices run over 1, 2, 3, 4, and lower case Latin indices run over $5, 6, \ldots, n+4$.

Now let us turn to the nonsymmetric metrization of the bundle $\mathbf{P}$. According to Kalinowski (1983c–e, 1984b), we have

$$\gamma_{AB} = \left(\begin{array}{c|c} g_{\alpha\beta} & 0 \\ \hline 0 & \rho^{2}l_{ab} \end{array}\right) \tag{7}$$

where $l_{ab} = h_{ab} + \mu K_{ab}$, $\rho = \rho(x) > 0$ is a scalar field on a space-time E, $\lambda = 2$ [$\lambda = (2/c^{2})\sqrt{G_N}$ in the cgs system of units], and

$$h_{ab} = C^{c}_{ad}C^{d}_{bc} \tag{8}$$

is a Killing–Cartan tensor on G, C^{c}_{ab} are structure constants of the Lie algebra of the group G,

$$[X_{a}, X_{b}] = C^{c}_{ab}X_{c} \tag{9}$$

$$K_{ab} = C^{c}_{ab}\,\mathrm{Tr}[(X_{c})^{2}] \tag{10}$$

is a skew-symmetric tensor on G (Kalinowski, 1983c–e; 1984b), and μ is a dimensionless real constant.

On the bundle $\mathbf{P}$ we define a 2-form of the curvature for the connection w,

$$\Omega = \text{hor}\, dw = dw + \tfrac{1}{2}[w, w] = \tfrac{1}{2}H^{a}_{\mu\nu}\theta^{\mu} \wedge \theta^{\nu}X_{a} \tag{11}$$

This form satisfies the Bianchi identity

$$\text{hor}\ d\Omega = 0 \tag{12}$$

Now we define on **P** a connection ω^A_B such that

$$D\gamma_{A+B-} = D\gamma_{AB} - \gamma_{AD} Q^D_{BC}(\Gamma)\theta^C = 0 \tag{13}$$

where $\omega^A_B = \Gamma^A_{BC}\theta^C$. Here D is the exterior covariant derivative with respect to the connection ω^A_B and $Q^D_{BC}(\Gamma)$ is the tensor of torsion for the connection ω^A_B. According to Kalinowski (1984b), we have

$$\omega^A_B = \left(\begin{array}{c|c} \pi^*(\bar{\omega}^\alpha_\beta) - \rho^2 l_{db} g^{\mu\alpha} L^d_{\mu\beta}\theta^b & L^a_{\beta\gamma}\theta^\gamma + \rho^{-1} g_{\beta\delta}\tilde{g}^{(\delta\gamma)}\rho_{,\gamma}\theta^a \\ \hline l_{bd} g^{\alpha\beta}(2H^d_{\gamma\beta} - L^d_{\gamma\beta})\theta^\gamma - \rho\tilde{g}^{(\alpha\beta)}\rho_{,\beta} l_{bc}\theta^c & \rho^{-1} g_{\delta\beta}\tilde{g}^{(\delta\gamma)}\rho_{,\gamma}\delta^a_b\theta^b + \tilde{\omega}^a_b \end{array}\right) \tag{14}$$

where

$$L^d_{\gamma\beta} = -L^d_{\beta\gamma} \tag{15}$$

$$l_{dc} g_{\mu\beta} g^{\gamma\mu} L^d_{\gamma\alpha} + l_{cd} g_{\alpha\mu} g^{\mu\gamma} L^d_{\beta\gamma} = 2 l_{cd} g_{\alpha\mu} g^{\mu\gamma} H^d_{\beta\gamma} \tag{16}$$

$$\tilde{\omega}^a_b = \tilde{\Gamma}^a_{bc}\theta^c \tag{17}$$

Here

$$\tilde{g}^{(\alpha\beta)} g_{(\alpha\gamma)} = \delta^\beta_\gamma \tag{18}$$

is an inverse tensor for $g_{(\alpha\beta)}$. Also,

$$l_{db}\tilde{\Gamma}^d_{ac} + l_{ad}\tilde{\Gamma}^d_{cb} = \tfrac{1}{2} l_{ad} C^d_{bc} \tag{19}$$

$$\tilde{\Gamma}^d_{ac} = -\tilde{\Gamma}^d_{ca} \tag{20}$$

$$\tilde{\Gamma}^d_{ad} = 0 \tag{21}$$

According to Kalinowski (1984b), we define on **P** a second connection

$$W^A_B = \omega^A_B - \frac{4}{3(n+2)}\delta^A_B \bar{W} \tag{22}$$

According to Kalinowski (1984b), we write down the Moffat–Ricci curvature scalar for W^A_B and we get

$$(-g)^{1/2} R(W) = (-g)^{1/2}\{\bar{R}(\bar{W}) + e^{(n+2)\Psi}\tilde{R}(\tilde{\Gamma}) + 8\pi e^{-(n+2)\Psi} L_{\text{YM}} + L_{\text{scal}}(\Psi)\} + \partial_\mu K^\mu \tag{23}$$

where $R(W)$ is the Moffat-Ricci curvature scalar for the connection W^A_B, $\tilde{R}(\tilde{\Gamma})$ is the Moffat-Ricci curvature scalar for the connection $\tilde{\omega}^a_b$, and

$$L_{\mathrm{YM}} = -\frac{1}{8\pi} l_{ab}(2H^a H^b - L^{a\mu\nu} H^b_{\mu\nu}) \tag{24}$$

$$H^a = g^{[\mu\nu]} H^a_{\mu\nu} \tag{25}$$

$$L^{a\mu\nu} = g^{\alpha\mu} g^{\beta\nu} L^a_{\alpha\beta} \tag{26}$$

$R(W)$ is the Lagrangian in the nonsymmetric Jordan-Thiry theory, and L_{YM} plays the role of the Lagrangian for the Yang-Mills field. $\tilde{R}(\tilde{\Gamma})$ plays the role of the cosmological constant and $\bar{R}(\bar{W})$ is the Lagrangian of the gravitational field in the nonsymmetric theory of gravitation. $L_{\mathrm{scal}}(\Psi)$ plays the role of the Lagrangian for the scalar field Ψ.

$L^a_{\mu\nu}$ plays the role of the second tensor of the Yang-Mills (gauge) field strength (Kalinowski, 1982b, 1982e, f, 1984b).

Equation (16) expresses the relationship between tensors $H^a_{\mu\nu}$ and $L^a_{\mu\nu}$. This relationship is linear with respect to $H^a_{\mu\nu}$ and $L^a_{\mu\nu}$ and nonlinear with respect to $g_{\alpha\beta}$. We have

$$L_{\mathrm{scal}}(\Psi) = (m\tilde{g}^{(\gamma\nu)} + n g^{[\mu\nu]} g_{\delta\mu} \tilde{g}^{(\delta\gamma)}) \Psi_{,\nu} \cdot \Psi_{,\gamma} \tag{27}$$

where

$$m = l^{[dc]} l_{[dc]} - n(n-1)$$

The field Ψ is connected with the field ρ via

$$\rho = e^{-\Psi} \tag{28}$$

This field is related to the effective gravitational "constant," which now is a function of space-time.

In the electromagnetic case $G = U(1)$ we have similarly

$$(-g)^{1/2} R(W) = (-g)^{1/2} \{\bar{R}(\bar{W}) + e^{-3\Psi}[2(g^{[\mu\nu]} F_{\mu\nu})^2 - H^{\mu\nu} F_{\mu\nu}] + L_{\mathrm{scal}}(\Psi)\} + \partial_\mu K^\mu \tag{29}$$

where

$$L_{\mathrm{scal}}(\Psi) = g^{[\nu\mu]} g_{\delta\nu} \tilde{g}^{(\alpha\delta)} \Psi_{,\mu} \cdot \Psi_{,\alpha} \tag{30}$$

is a Lagrangian for the scalar field Ψ. We have

$$H^{\mu\nu} = g^{\alpha\mu} g^{\beta\nu} H_{\alpha\beta} \tag{31}$$

$$g_{\delta\beta} g^{\gamma\delta} H_{\gamma\alpha} + g_{\alpha\delta} g^{\delta\gamma} H_{\beta\gamma} = 2 g_{\alpha\delta} g^{\delta\gamma} F_{\beta\gamma}, \qquad H_{\beta\gamma} = -H_{\gamma\beta} \tag{32}$$

$F_{\mu\nu}$ is the strength of the electromagnetic field,

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{33}$$

where A_α is a 4-potential of the electromagnetic field. $H_{\mu\nu}$ is a second tensor of the electromagnetic field strength (Kalinowski, 1983e). The field Ψ is related to the field ρ by the formula (28) and the effective gravitational constant is expressed by

$$G_{\text{eff}} = G_{\text{N}} e^{-3\Psi} \tag{34}$$

If we put $\Psi = 0$, we get the nonsymmetric Kaluza-Klein theory (Kalinowski, 1983c–e, 1984b).

From the Palatini variational principle for (23) we get the field equations (Kalinowski, 1984b)

$$\bar{R}_{\mu\nu}(\bar{W}) - \tfrac{1}{2} g_{\mu\nu} \bar{R}(\bar{W}) = 8\pi K [\overset{\text{gauge}}{T}_{\mu\nu} + \overset{\text{scal}}{T}_{\mu\nu}(\Psi) + g_{\mu\nu}\Phi] \tag{35}$$

$$\mathbf{g}^{[\mu\nu]}_{,\nu} = 0; \qquad g_{\mu\nu,\sigma} - g_{\zeta\nu} \bar{\Gamma}^{\zeta}_{\mu\sigma} - g_{\mu\zeta} \bar{\Gamma}^{\zeta}_{\sigma\nu} = 0 \tag{36}$$

$$\begin{aligned} \overset{\text{gauge}}{\nabla}_{\mu}(\mathbf{L}^{a\alpha\mu}) = {} & 2g^{[\alpha\beta]} \overset{\text{gauge}}{\nabla}_{\beta}(g^{[\mu\nu]} H^{a}_{\mu\nu}) \\ & -(n+2)\partial_{\beta}\Psi[\mathbf{L}^{a\beta\alpha} - 2\mathbf{g}^{[\beta\alpha]}(g^{[\mu\nu]} H^{a}_{\mu\nu})] \end{aligned} \tag{37}$$

$$\begin{aligned} & [(n+2m)\tilde{g}^{(\alpha\mu)} - n g^{\nu\mu} g_{\delta\nu} \tilde{g}^{(\alpha\delta)}] \frac{\partial^2 \Psi}{\partial x^{\alpha}\, \partial x^{\mu}} \\ & + \frac{1}{(-g)^{1/2}} \partial_{\mu} \left\{ (-g)^{1/2} \left[n\tilde{g}^{(\mu\alpha)} - \frac{n}{2} g_{\delta\nu} (g^{\nu\alpha} \tilde{g}^{(\mu\delta)} + g^{\nu\mu} \tilde{g}^{(\alpha\delta)}) \right.\right. \\ & \left.\left. - 2m\tilde{g}^{(\mu\alpha)} \right] \right\} \frac{\partial \Psi}{\partial x^{\alpha}} - 8(n+2)\pi\, e^{-(n+2)\Psi} (L_{\text{YM}} - 2\Phi) \\ = {} & 0 \end{aligned} \tag{38}$$

where

$$\begin{aligned} \overset{\text{gauge}}{T}_{\alpha\beta} = {} & -\frac{l_{ab}}{4\pi} \{ g_{\gamma\beta} g^{\tau\zeta} g^{\varepsilon\gamma} L^{a}_{\zeta\alpha} L^{b}_{\tau\varepsilon} - 2g^{[\mu\nu]} H^{(a}_{\mu\nu} H^{b)}_{\alpha\beta} \\ & - \tfrac{1}{4} g_{\alpha\beta} [L^{a\mu\nu} H^{b}_{\mu\nu} - 2(g^{[\mu\nu]} H^{a}_{\mu\nu})(g^{[\gamma\delta]} H^{b}_{\gamma\delta})] \} \end{aligned} \tag{39}$$

is the energy-momentum tensor for the gauge (Yang-Mills) field,

$$\overset{\text{scal}}{T}_{\alpha\beta}(\Psi) = \frac{e^{(n+2)\Psi}}{16\pi} \left\{ g_{\kappa\alpha} g_{\omega\beta} + g_{\omega\alpha} g_{\kappa\beta}) \tilde{g}^{(\gamma\kappa)} \tilde{g}^{(\nu\omega)} \right.$$

$$\times\left[\frac{n}{2}(g^{\zeta\mu}g_{\nu\zeta}-\delta^{\mu}_{\nu})\Psi_{,\mu}+m\Psi_{,\nu}\right]\Psi_{,\gamma}$$

$$\left.-g_{\alpha\beta}(m\tilde{g}^{(\gamma\nu)}+ng^{[\mu\nu]}g_{\delta\mu}\tilde{g}^{(\gamma\delta)})\Psi_{,\nu}\cdot\Psi_{,\gamma}\right\} \quad (40)$$

is the energy-momentum tensor for the scalar field Φ, and

$$\Phi=e^{2(n+2)\Psi}\frac{\tilde{R}(\tilde{\Gamma})}{16\pi}=e^{2(n+2)\Psi}\rho(\mu) \quad (41)$$

is the cosmological term. If $\Psi=0$, $\rho=1$,

$$K=e^{-(n+2)\Psi}=G_{\text{eff}} \quad (42)$$

We put $G_N=C=1$. In these equations $\overset{\text{gauge}}{\nabla}_{\mu}$ means gauge derivative;

$$\mathbf{L}^{a\mu\nu}=(-g)^{1/2}L^{a\mu\nu};\qquad \mathbf{g}^{[\mu\nu]}=(-g)^{1/2}g^{[\mu\nu]} \quad (43)$$

[Kalinowski (1984b) for more details].

Thus we get a theory that unifies gravity, gauge fields, and scalar forces. The gravitational field in this theory is described by a nonsymmetric, real tensor $g_{\mu\nu}$ (and a scalar field Ψ), which connects it with Moffat's theory of gravitation [one of the most important alternative theories of gravitation; see Moffat (1982) for a review]. The nonsymmetric Kaluza-Klein (Jordan-Thiry) theory has been previously designed as a unification of Moffat's theory of gravitation and the electromagnetic (or Yang-Mills) field. However, there are some differences. First, Moffat and his co-workers make extensive use of the theory of Einstein and Strauss (1946) in a hypercomplex-Hermitian version (Kunstatter et al., 1983), but not that of Einstein and Kaufman (1954). The Einstein-Strauss theory cannot be extended in any simple way to higher dimensions, even in the five-dimensional (electromagnetic) case. It is also a hard task to incorporate spin sources in the Einstein-Strauss theory. In both cases, we face a fundamental physical problem. The Lagrangian becomes hypercomplex (not real). In the present case these problems do not arise, because everything is real. In the case of the nonsymmetric Jordan-Thiry theory, one effectively gets the scalar-tensor theory of gravitation in the nonsymmetric version. The scalar field behaves very well in the linear approximation. It has been proved (Kalinowski and Mann, 1983) that one can avoid tachyons and ghosts in the particle spectrum of the theory (if one puts $m>0$). In the case of classical Jordan-Thiry theory, the scalar field is a ghost (a negative kinetic energy). This new version of the Kaluza-Klein theory is capable of removing singularities from the solution of coupled gravitational and Yang-Mills equations even in the case of spherical symmetry. Such solutions have been found in the electromagnetic case (Kalinowski and Kunstatter, 1984; Mann, 1984). It is well known that in the case of Einstein-Maxwell equations one cannot get any nonsingular, localizable, stationary solutions (the so-called Hilbert-Levi-Civita-Thiry-Einstein-Lichnerowicz-Pauli theorem) (Hilbert, 1916; Levi-Civita, 1917; Lichnerowicz, 1939; Einstein and Pauli, 1943). This result has been recently extended to the case of non-Abelian gauge fields (Weder, 1982).

Recently, Mann (1984) found eight classes of spherically symmetric and stationary solutions in the nonsymmetric Kaluza-Klein theory. These solutions are more general than the result of Kalinowski and Kunstatter (1984) and some of them have no singularities in gravitational and electromagnetic fields. Some of these solutions possess a nonzero magnetic field and nonzero $g_{[23]}=f\neq 0$. The nonsingular solutions are parametrized by fermion charge l^2, electric charge Q, and a new constant u_0. This constant is related to $g_{[23]}$ in the same way that l^2 is related to $g_{[14]}$. It plays a similar role for $g_{[\mu\nu]}$ that a magnetic charge plays for $F_{\mu\nu}$. Note that the first exact solution found in Kalinowski and Kunstatter (1984) has no singularity in an electric field and a finite energy. However, it has a weak singularity in $g_{(\alpha\beta)}$. In this case one puts $g_{[23]}=0$. It seems that these solutions can be extended without any problems to the non-Abelian case.

Thus one can look for models of elementary particles as exact solutions of field equations.

In the theory there are two field strengths for the electromagnetic (Yang-Mills) field: $F_{\mu\nu}$ and $H_{\mu\nu}$ ($H^a_{\mu\nu}$ and $L^a_{\mu\nu}$). The first is built from ($\mathbf{E}$, $\mathbf{B}$) [($\mathbf{E}^a$, $\mathbf{B}^a$)] and the second from ($\mathbf{D}$, $\mathbf{H}$) [($\mathbf{D}^a$, $\mathbf{H}^a$)]. The relations between both tensors are given by equations (16) and (32).

According to current ideas (Kogut, 1983; Lee, 1979, 1981) the confinement of color could be connected to the dielectricity of the vacuum (dielectric model of confinement). Due to the so-called antiscreening mechanism, the effective dielectric constant is equal to zero. This means that the energy of an isolated charge goes to infinity. Now there are also so-called classical dielectric models of confinement (Lehman and Wu, 1983). The confinement is induced by a special kind of dielectricity of the vacuum such that $\mathbf{E}\neq 0$ and $\mathbf{D}=0$ ($\mathbf{E}^a\neq 0, \mathbf{D}^a=0$). In this case there is no distribution of charge. This is similar to an electric type of Meissner effect.

It is easy to see that in the present case (the nonsymmetric Kaluza-Klein theory) the dielectricity is induced by the nonsymmetric tensors $g_{\mu\nu}$ and l_{ab}. If $g_{[\mu\nu]}=0$, $\mathbf{D}=\mathbf{E}$ and $\mathbf{B}=\mathbf{H}$.

The gravitational field described by the nonsymmetric tensor $g_{\mu\nu}$ behaves as a medium for an electromagnetic field (gauge field). The condition $\mathbf{E}\neq 0$, $\mathbf{D}=0$ ($\mathbf{E}^a\neq 0$, $\mathbf{D}^a=0$) can be satisfied in the axial, stationary case for $F_{\mu\nu}$, $H_{\mu\nu}$ ($H^a_{\mu\nu}$, $L^a_{\mu\nu}$), and $g_{\mu\nu}$. Thus it is interesting to find an exact solution with axial symmetry for the nonsymmetric Kaluza-Klein theory with fermion sources for $G=SU(3)_c$. This could provide a model of a hadron.

The axially symmetric, stationary case seems to be very interesting from a more general point of view. In General Relativity one has very peculiar properties of stationary, axially symmetric solutions of the Einstein-Maxwell equations. These solutions describe the gravitational and electromagnetic fields of a rotating charged mass. Thus, one gets the magnetic field component. Asymptotically (these solutions are asymptotically flat) the magnetic field behaves as a dipole field. One can calculate the gyromagnetic ratio at infinity, i.e., the ratio of the magnetic dipole moment and the angular momentum moment. It is worth noticing that one gets the anomalous gyromagnetic ratio (Kramer et al., 1980), i.e., the gyromagnetic ratio for an electron (for a charged Dirac particle). One cannot interpret the Kerr-Newman solution as a model of the fermion, for there is a singularity. In the nonsymmetric Kaluza-Klein theory one can expect completely nonsingular solutions (Kalinowski and Kunstatter, 1984; Mann, 1984). One can also expect the asymptotic behavior of the Einstein-Maxwell theory. Thus it seems that one will probably get the solutions with anomalous gyromagnetic ratio. Such a solution could be treated as a model (classical) of a spin-$\frac{1}{2}$ particle. In the non-Abelian case $[G=SU(3)_c\times U(1)_{\mathrm{em}}]$ this could provide a model of a charged baryon (i.e., proton), where the skewon field $g_{[\mu\nu]}$ induces a confinement of color. In this way, the skewon field $g_{[\mu\nu]}$ plays a double role: (1) additional gravitational interactions (from Moffat's theory of gravitation), and (2) a strong interaction field connected to the confinement problem.

It has been proved by Mann and Moffat (1982a,b) that the skewon field $g_{[\mu\nu]}$ has zero spin. In a linear approximation it is the so-called generalized Maxwell field (an Abelian gauge field). Thus it is natural to expect an exchange of some spin-zero particles in the nucleon-nucleon potential for low and intermediate energies. Such particles are not observed. However, one cannot fit experimental data for the nucleon-nucleon interaction without the mysterious σ- (spin-zero) particles (see, for example, Bryan and Scott, 1964; Brown, 1972; Mau Vinh, R., 1978; Rho, 1984).

It happens that two such particles are needed in order to fit the data. In the present proposal, they are connected to the skewon field $g_{[\mu\nu]}$ and to the scalar field Ψ from the nonsymmetric Jordan-Thiry theory. The reason such particles are not detected directly now seems clear. They are confined, because they are actually a cause of confinement. The scalar field from the nonsymmetric Jordan–Thiry theory induces an additional dielectricity of the vacuum [see the Lagrangians for the scalar field Ψ and for the Yang–Mills field in equations (23), (24), and (27)]. Note that a function of the scalar field Ψ appears as a factor before the Yang–Mills Lagrangian in equation (23). This has some important consequences: the effective gravitational "constant" depends on Ψ and in the flat space limit $g_{\mu\nu} = \eta_{\mu\nu}$ the Lagrangian resembles the bosonic part of the soliton bag model Lagrangian if one puts

$$e^{-10\Psi} = 2\left(1 - \frac{\sigma}{\sigma_0}\right); \qquad \sigma_0 = \text{const} \tag{44}$$

for $n = 8$, $G = SU(3)$ (De Tar and Donoghue, 1983; Goldflam and Wilets, 1982). One finds

$$\Psi = -\frac{1}{10}\ln\left(1 - \frac{\sigma}{\sigma_0}\right) - \frac{\ln 2}{10} \tag{45}$$

and in the flat space limit one easily gets

$$L = -\frac{1}{4}\left(1 - \frac{\sigma}{\sigma_0}\right)(h_{ab} + \mu^2 K^c_b K_{ca}) H^a_{\mu\nu} H^{b\mu\nu} + \frac{\sigma_0 \rho(\mu)}{16\pi(\sigma_0 - \sigma)} + \frac{m\sigma_0^2}{100(\sigma_0 - \sigma)^4}\eta^{\mu\nu}\sigma_{,\mu}\cdot\sigma_{,\nu} \tag{46}$$

The full Lagrangian (23) is more general and it contains a gravitational field.

Friedberg and Lee (1978) consider the soliton bag model with a more general factor $K(\sigma)$,

$$L = -\tfrac{1}{4}K(\sigma) h_{ab} H^{a\mu\nu} H^b_{\mu\nu} - \tfrac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - U(\sigma) \tag{47}$$

They consider that the scalar field σ is a new dynamical field with self-interaction given by $U(\sigma)$. The quantity K is a dielectric constant which depends on σ. It is interesting to observe the many similarities between (47) and the Lagrangian from the nonsymmetric Jordan–Thiry theory, i.e., (23). Thus, in the present model one has in the flat space limit an effective dielectric constant

$$K_{\text{eff}} = 4e^{-10\Psi} \tag{48}$$

It is interesting to notice that the scalar field Ψ enters into the effective gravitational "constant" and into the effective dielectric "constant" in the flat space limit.

In a full nonsymmetric Jordan–Thiry theory (curved non-Riemannian space-time) one has the following symmetry for the scalar field (Kalinowski, 1983d, 1984b):

$$\Psi \to \Psi' = f(\Psi) \tag{49}$$

where f is an arbitrary function. In this way the formulas (44) and (48) can be treated as transformations for a scalar field in the nonsymmetric Jordan–Thiry theory. Thus it is possible to connect a bosonic part of some soliton bag model Lagrangians via equation (49) in the nonsymmetric Jordan–Thiry theory. In this way one can see some possibilities of connecting gravitational and strong interactions via the nonsymmetric Kaluza–Klein (Jordan–Thiry) theory. This is somewhat in the spirit of an idea of strong gravity (Isham et al., 1978). In this approach, there are two metric (symmetric) tensors. It is easy to see that in the nonsymmetric Kaluza–Klein (Jordan–Thiry) theory there are two metric (symmetric) tensors $g_{(\alpha\beta)}$ and $f_{\alpha\beta}$ such that

$$f_{\alpha\beta} g^{(\alpha\gamma)} = \delta^{\gamma}_{\beta}; \qquad g_{\alpha\beta} g^{\alpha\gamma} = g_{\beta\alpha} g^{\gamma\alpha} = \delta^{\gamma}_{\beta} \tag{50}$$

and it is easy to see that if $g_{[\alpha\beta]} = 0$, then $f_{\alpha\beta} = g_{(\alpha\beta)}$.

Thus I propose the Lagrangian of the nonsymmetric Jordan–Thiry theory as the bosonic part of the Lagrangian of strong interactions. Why? It seems that something is missing in the QCD Lagrangian. One has the following objectives: (1) σ-particles (which were mentioned earlier), and (2) an exact solution with color radiation (this means color at infinity—no confinement) as found by Tafel and Trautman (1983).

Thus it seems that the QCD Lagrangian is incomplete in the bosonic part. In the present proposal, the QCD Lagrangian is replaced by the Lagrangian from nonsymmetric, non-Abelian Jordan–Thiry theory for $G = SU(3)_c$. In this way one can get a dielectric model of confinement and a soliton bag model-like Lagrangian (De Tar and Donoghue, 1983; Goldflam and Wilets, 1982; Friedberg and Lee, 1978).

Thus I propose the following program of investigation:

1. Find exact solutions of the nonsymmetric Kaluza–Klein and Jordan–Thiry theories in Abelian and non-Abelian cases with and without fermion sources in the case of spherical and axial symmetry, using inverse scattering, and the Riemann invariants method.
2. Find an effective interaction of two axially symmetric solutions exactly, or, using some numerical methods in the case of $G = SU(3)_c$, with fermion sources. This could be similar to the nucleon–nucleon interaction in the Skyrme model. The solutions sould be treated as particles using a collective coordinate method.
3. Find wavelike solutions of the field equations in the Abelian and non-Abelian cases. In the electromagnetic case, this could offer a

solution that could be treated as a kind of electromagnetogravitational wave (nonlinear wave) with nontrivial interactions between all fields. The objective of this hope is related to points 4 and 5 in the list of "interference effects" in Section 2 (recall that the displacement current in the classical Maxwell equations leads to the nontrivial interaction between the electric and magnetic fields—the *raison d'être* of the wave solutions for the Maxwell equations; however, this is only a historical remark). By a nontrivial interaction, I mean that the flow of energy is possible from one field to the second in a quasiperiodic way.

There are also some proposals concerning cosmology:

1. Find a Bianchi type I cosmological solution in the nonsymmetric Kaluza-Klein theory with material sources (Kalinowski, 1984c). One expects completely nonsingular solutions in the presence of an electromagnetic field.
2. Find a new (or old) inflationary scenario for the Universe from the nonsymmetric, non-Abelian Kaluza-Klein theory. It has been shown (Kalinowski, 1983f) that one can get a Higgs potential with two critical points from the nonsymmetric Kaluza-Klein theory. This offers phase transitions in early cosmology and could give Guth's new (or old) inflationary scenario without the Coleman-Weinberg theory.

It is also interesting to do some research under the formal structure of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories:

1. A rigorous treatment of the nonsymmetric tensor $l_{ab} = h_{ab} + \mu K_{ab}$ defined on the algebra of matrices (enveloping algebra of the Lie algebra of the gauge symmetry group).
2. An extension of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories including supergravity and supersymmetry [some ideas on how to do this can be found in Kalinowski (1984c) and Goldflam and Wilets (1982)].
3. Studies under a spontaneous compactification of an n-dimensional submanifold of an $(n+4)$-dimensional manifold with Einstein geometry (a global or/and local compactification).

3. CONCLUSIONS

In this paper I have proposed the Lagrangian of the nonsymmetric, non-Abelian Jordan-Thiry theory as the bosonic part of the Lagrangian of strong interactions. In this way the QCD Lagrangian would be extended, including the skewon field $g_{[\mu\nu]}$ and the scalar field Ψ. Both fields $g_{[\mu\nu]}$ and

Ψ play double roles: (1) as a part of gravitational interactions, and (2) as a part of a strong interaction field. The existence of $g_{[\mu\nu]}$ and Ψ could explain (in principle) the σ-particles in a nucleon–nucleon potential and a confinement of color via the classical dielectric model of confinement. It is possible on the level of the nonsymmetric Jordan–Thiry theory to connect some soliton bag models via transformation of the scalar field Ψ.

I have proposed a program of research that consists in finding exact solutions in this theory. These solutions could be treated as models of particles [generalized skyrmions (Skyrme, 1961; Adkins et al., 1982)]. The present approach seems more realistic, because it includes the Lagrangian gauge and gravitational fields. In the Skyrme model one has to deal with an effective model of strong interactions. This model, despite many spectacular successes, has some problems, for example, a mass difference between the nucleon and Δ^{2+} particle. Moreover, the interactions between two skyrmions can give a qualitatively good description of the nucleon–nucleon potential (Rho, 1984). In this way it is possible to approach some classical nuclear phenomenology as in Thomas (1982).

One could search for axially symmetric, stationary solutions in the nonsymmetric Kaluza–Klein (Jordan–Thiry) theory using the formalism presented in Mihich (1983). Finally, some of Witten's (1981) ideas can be employed for the nonsymmetric Kaluza–Klein (Jordan–Thiry) theory.

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